Fractal Analysis of Fluidized Particle Behavior in Liquid-Solid Fluidized Beds

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Liquid-solid fluidized beds have been adopted widely for catalytic liquid-phase reactions, separation and recovery of materials with ion exchange resin, adsorption, sedimentation, and wastewater treatment. Their unique features are the effective liquid-solid contact and the high rates of heat and mass transfer (Morooka et al., 1980; Muroyama et al., 1986; Kang and Kim, 1987). Nevertheless, the performance of a liquid-solid fluidized bed as a contactor or reactor is influenced appreciably by various factors, such as the flow regime (Volpicelli et al., 1966; Kang and Kim, 1988), the motion of fluidized particles (Handley et al., 1966; Kmiec, 1978; Yutani et al., 1983), and voidage distribution (Fan et al., 1985); this gives rise to highly complicated, nonlinear and stochastic behavior of the bed. While some attempts had been made to stochastically analyze and model such behavior, they were mostly based on the Markovian assumptions and notion of classical Brownian motion (Yutani et al., 1982, 1983; Yutani and Fan, 1985; Fan et al., 1990a; Kang et al., 1990).

Our recent works on multiphase flow systems (Fan et al., 1989, 1990b) have indicated that the concept of fractional Brownian motion (fBm), which can be characterized by the Hurst exponent, H, may be applicable to the analysis of liquid-solid fluidized beds. The model based on this concept has been proposed by Mandelbrot and van Ness (1968) to identify the long-term correlation in a time series, which is self-affine. It has been postulated that the rescaled range (R/S) analysis can be a means of estimating H for a given time series (Mandelbrot and Wallis, 1969a,b; Feder, 1988).

In this work, pressure fluctuations in a liquid-solid fluidized bed have been examined by resorting to the R/S analysis; the effects of fluidized particle size, axial position, and liquid flow rate on the Hurst exponent, H, or the local fractal dimension, d_{FL}, have been examined. The results have revealed that the pressure fluctuations in the bed, exhibiting long-term correlation, essentially reflect the behavior of fluidized particles, and thus, these particles can be considered to undergo fractional Brownian motion.

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Experimental Studies

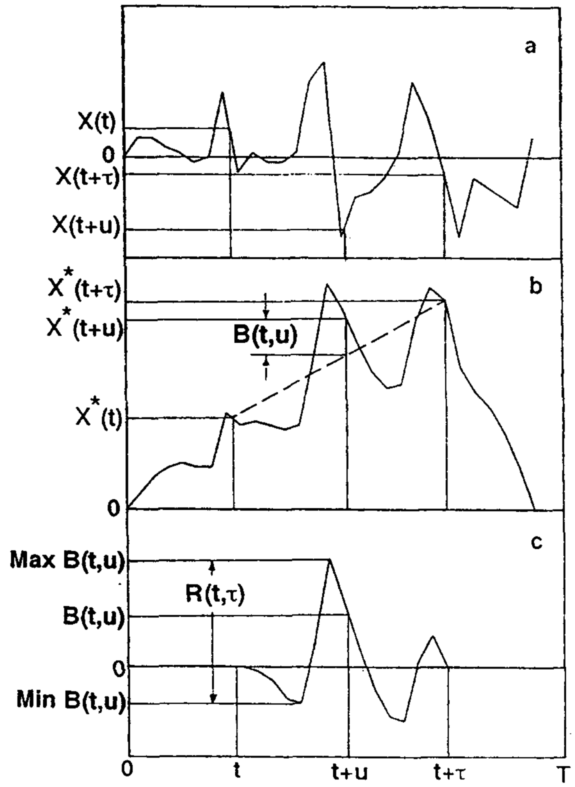
Experiments were carried out in a column with an ID of 0.0508 m and a height of 1.5 m. A wire net (opening of 0.354 $\times 10^{-3}$ m) and a packed bed of alumina beads served as the liquid distributor. The fluidized solid particles were glass beads with a density of 2,500 kg/m^3. The diameter of the glass beads was either 0.001 or 0.006 m.

Pressure taps were installed on the wall of the column at six different heights from the distributor. Each pressure tap was connected to one of the two input channels of a differential pressure transducer (Enterprise Model CJ3D), which produced an output voltage proportional to the pressure difference between the two channels. The remaining channel was exposed to the atmosphere.

Signals were processed by an oscilloscope, a personal computer (Zenith 386), and a mainframe computer (IBM 3084). The voltage-time signal, corresponding to the pressure-time signal, from the transducer was fed to the recorder at the selected sampling rate of 0.005 s. A typical sample comprised 4,000 points. This combination of the sampling rate and sample length ensured that the full spectrum of hydrodynamic signals were captured from the liquid-solid fluidized bed; the signals were processed off-line.

Results and Discussion

The experimental results indicated that, in general, the amplitude of pressure fluctuations in the liquid-solid fluidized beds of glass beads, 0.006 m in diameter, was larger than that in the beds of glass beads, 0.001 m in diameter, under comparable operating conditions and at comparable bed locations. In other words, the larger the particle diameter, the greater the amplitude of pressure fluctuations. The patterns of these pressure fluctuations, which were observed to be varied and



$$S^2(t,\tau)=\frac{1}{\tau}\sum_{u=t+1}^{t+\tau}X^2(u)-\left[\frac{1}{\tau}\sum_{u=t+1}^{t+\tau}X(u)\right]^2$$

$$X^*(t)=\sum_{u=1}^{t}X(u)$$

$$\frac{1}{\tau}[X^*(t+\tau)-X^*(t)]$$

$$B(t,u)=[X^*(t+u)-X^*(t)]-\frac{u}{\tau}[X^*(t+\tau)-X^*(t)]$$

$$R(t,\tau)=\underset{0\le u\le\tau}{\mathrm{Max}}\,B(t,u)-\underset{0\le u\le\tau}{\mathrm{Min}}\,B(t,u)$$

$$\frac{R(t,\tau)}{S(t,\tau)}\propto\tau^H\ ;\ d_{FL}=2-H\ ,\ 0<H<1$$

Figure 1. Construction of the sample range, $R(t, \tau)$.

irregular, were examined by means of the rescaled range, R/S, analysis, thereby yielding the Pox diagrams; eventually the Hurst exponents were recovered from the diagrams. The procedure for estimating the Hurst exponent is briefly illustrated in Figure 1; the details of R/S analysis is discussed elsewhere (Fan et al., 1990b).

Figure 2 illustrates the resultant Pox diagrams for the particles, 0.001 m in diameter, for two cases. The rescaled range (R/S) for either case correlates linearly with the time lag, τ, in the entire range to yield a high correlation coefficient, r, of 0.99; the value of the Hurst exponent, H, recovered is 0.99. Nevertheless, the rescaled range in each plot overlaps or becomes compact at τ_o of approximately 10 s. Figure 3 illustrates the Pox diagrams for the particles, 0.006 m in diameter, again for two cases. The rescaled range for either case in this figure correlates linearly with the time lag, τ, to yield a correlation coefficient, r, of 0.98 or 0.97, but only up to the break at τ_o of approximately 4 s: the value of the Hurst exponent, H, recovered is 0.77. The overlapping and/or the break of the rescaled range at the time lag, τ_o, is attributable to the presence of the dominant periodic component of pressure fluctuations (Mandelbrot and Wallis, 1969b). Our results show that the smaller the particle diameter, the more persistent the stochastic component as signified by the larger value of H. Moreover, the smaller the particle diameter, the lower the frequency of the dominant periodic component as signified by the larger value of τ_o (Fan et al., 1990a).

Figure 4 shows the effects of the liquid flow rate, V_L, on the reciprocal of the time lag corresponding to the break of the R/S curve, $1/\tau_o$, in an individual bed of glass beads, 0.006 m in diameter. This figure exhibits a trend similar to that observed earlier by plotting the dominant frequency of the pressure fluctuations, f_o, obtained from the spectral analysis instead of $1/\tau_o$ against the liquid flow rate, V_L (Fan et al., 1990a). We have postulated that this major frequency, f_o, exhibits a local maximum at an intermediate liquid flow rate, where the transition of the particle flow regime occurs from cluster circulation to essentially random motion. Our data appear to confirm this postulation as demonstrated in Figure 5. In this figure, the values of τ_o obtained from this R/S analysis are plotted against those obtained from the previous spectral analysis for the same set of data.

The stochastic or random component of a pressure fluctuation signal can be characterized by the Hurst exponent, H, or the local fractal dimension, d_{FL} (Fan et al., 1989, 1990b). The Hurst exponent, H, has been recovered from the slope of the Pox diagram. The magnitude of the Hurst exponents obtained in this work indicates that the particle motion in the liquid-solid fluidized bed exhibits significant long-term correlation or undergoes fractional Brownian motion. As illustrated in Figure 6, H or d_{FL} exhibits a maximum or a minimum, respectively, as the liquid flow rate is varied in an individual bed of glass beads, 0.001 or 0.006 m in diameter. The larger the value of H, the more persistent the pressure fluctuations; thus, it is plausible that the pressure fluctuations are less irregular at the intermediate liquid flow rate. This is attributable to the fact that the behavior of fluidized particles is less irregular at the intermediate liquid flow rate (Fan et al., 1990a) since the

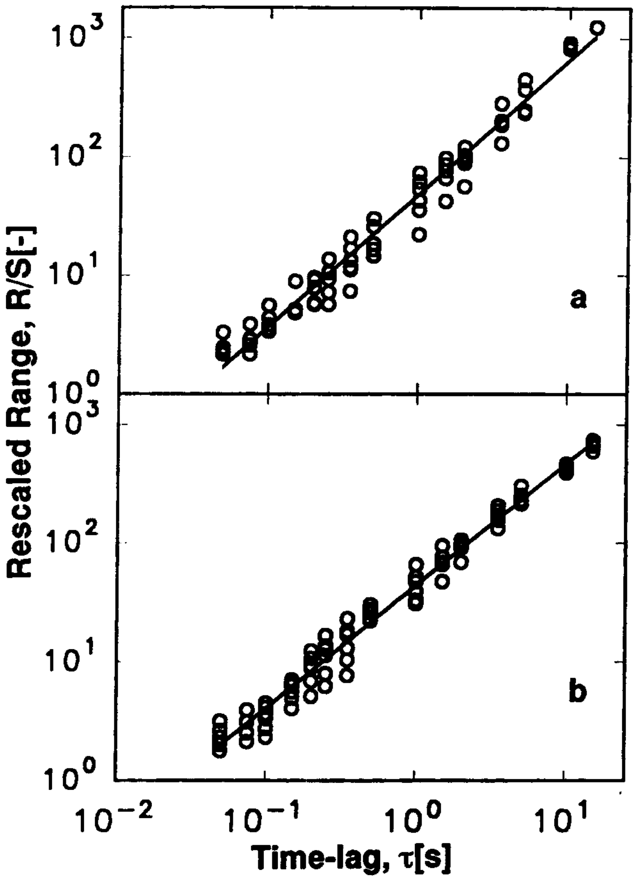


Figure 2. Pox diagram for pressure fluctuation signal in the beds of glass beads, 0.001 m in diameter.

	a	*b*
H_s (m)	0.26	0.13
H_p (m)	0.49	0.13
V_L (m/s)	0.08	0.10
H	0.99	0.99
r	0.99	0.99

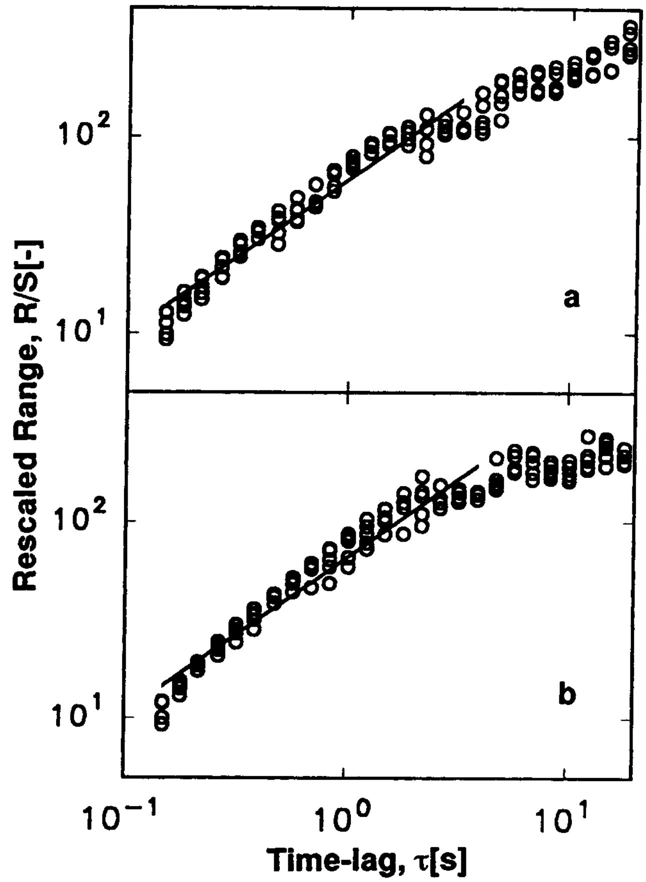


Figure 3. Pox diagram for pressure fluctuation signal in the beds of glass beads, 0.006 m in diameter.

	a	*b*
H_s (m)	0.14	0.14
H_p (m)	0.04	0.13
V_L (m/s)	0.10	0.16
H	0.77	0.77
r	0.98	0.97

motion of fluidized particles appears to be the major cause of pressure fluctuations in a liquid-solid fluidized bed. The liquid flow rate, V_l, corresponding to the maximum of H or the minimum of d_{FL} coincides closely with that where the flow regime of fluidized particles undergoes transition from cluster circulation to essentially random motion, as discussed in the preceding paragraph (Kang and Kim, 1988; Fan et al., 1990a). It is worth noting that cluster circulation is accompanied by microscopic random motion of individual particles within each cluster.

This work demonstrates that determining the Hurst exponent, H, facilitates the quantitative analysis of the time-series signal generated by such pressure fluctuations. Nonetheless, the results of such analysis should be interpreted in conjunction with those from the conventional methods and some of the modern tools, such as wavelet analysis (Gache et al., 1991).

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Notation

- d_{FL} = local fractal dimension
- f_o = dominant frequency, Hz
- H = Hurst exponent
- H_p = probe height, m
- H_s = static bed height, m
- r = correlation coefficient
- $R(t, \tau)$ = sample sequential range for lag τ
- $S^2(t, \tau)$ = variance
- t = time, s
- T = total available sample size
- $X(t)$ = time series

Greek letters

- τ = time-lag, s
- τ_o = time-lag corresponding to the break of the R/S curve, s

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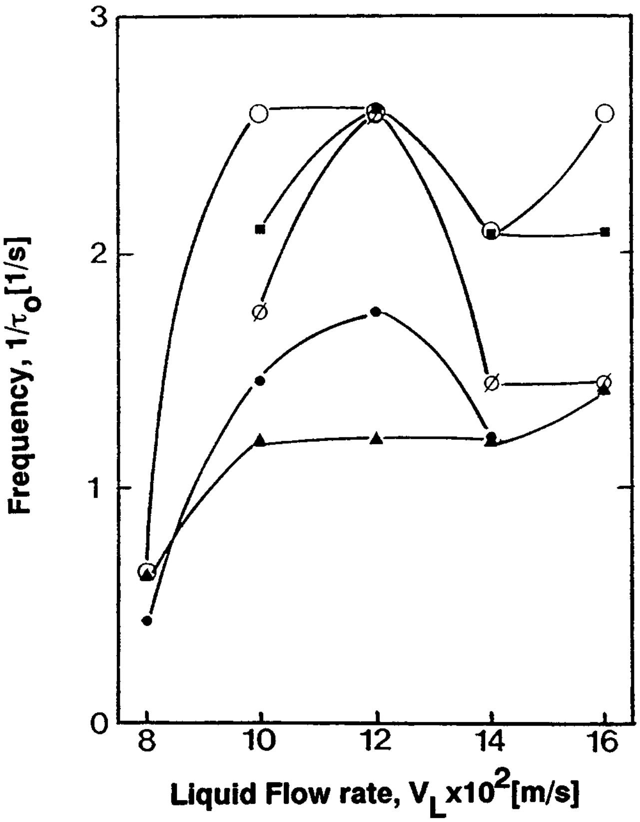

Figure 4. Effects of V_L on $1/\tau_o$ in the beds of glass beads, 0.006 m in diameter.

	○	ø	●	▲	■
H_s (m)	0.14	0.14	0.21	0.21	0.28
H_p (m)	0.04	0.13	0.13	0.22	0.04

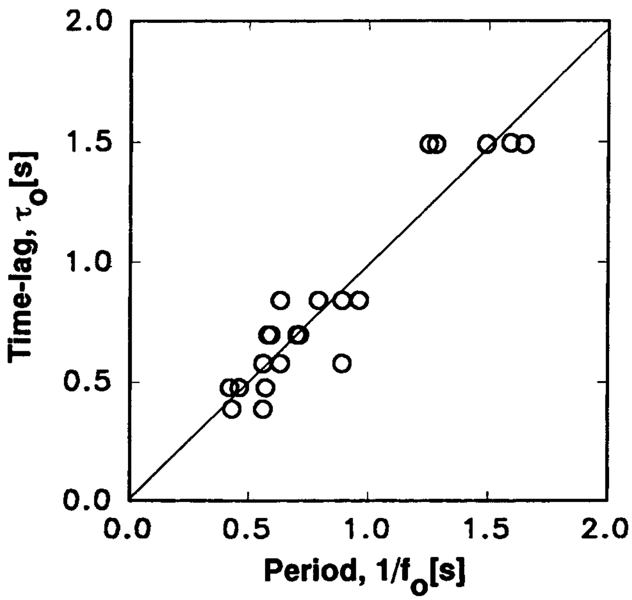

Figure 5. Values of τ_o obtained from the Pox diagram vs. those of $1/f_o$ from the spectral analysis in the beds of glass beads, 0.006 m in diameter: r = 0.94.

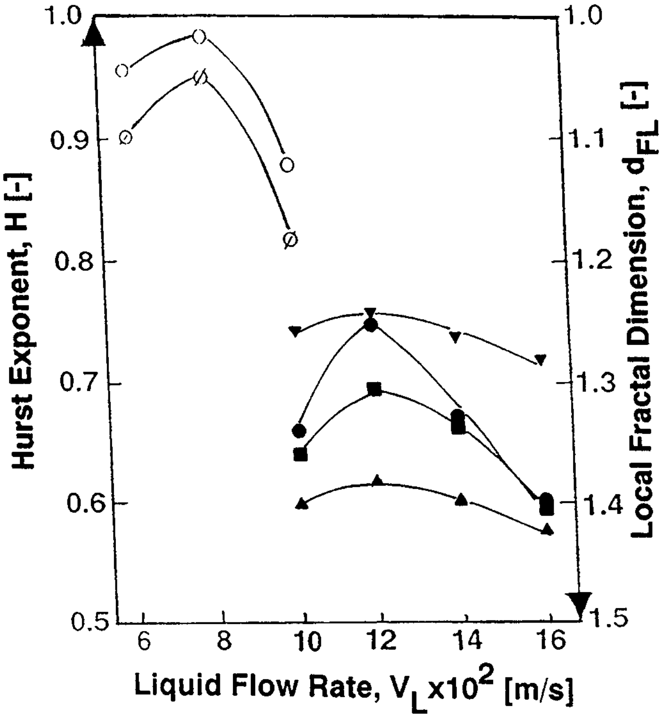

Figure 6. Effects of V_L on H or d_{FL} in the beds of glass beads of various sizes and amounts.

	○	ø	●	▲	■	▼
d_p (m)	0.001	0.001	0.001	0.006	0.006	0.006
H_s (m)	0.26	0.26	0.14	0.21	0.21	0.28
H_p (m)	0.04	0.22	0.13	0.13	0.22	0.22

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